Faculty of Science, Technology, Engineering and Mathematics
MST210 Mathematical methods, models and modelling

MST210

TMA 04 2019J

Covers Units 8, 9, 10 and 11 **Cut-off date 22 January 2020**

The module website gives details of how to submit assignments for this module. If you have a disability that makes it difficult for you to attempt any of the questions, then please contact your Student Support Team or your tutor for advice.

The marks allocated to each part of each question are indicated in brackets in the margin.

You may use Maxima to check answers, but it should not be used otherwise to answer TMA questions except where explicitly indicated. TMA answers must show full working to attract full marks.

In order to encourage you to present your solutions to the TMA questions in a good mathematical style, your tutor will comment on how you:

- use correct mathematical notation
- define any symbols that you introduce in formulating and solving a problem
- give references for standard formulas and derivations
- include comments and explanations within your mathematics
- explicitly state results and conclusions, giving answers to an appropriate degree of accuracy and interpreting answers in the context of the question
- draw diagrams and graphs.

These features are seen as being essential to complementing your mathematical skills. Your tutor will make comments on how well you achieve these objectives and give you guidance on how to satisfy them.

Five of the marks for this TMA will be allocated to the way you write your solutions. It is expected that most students will receive the majority of these presentation marks; such marks are included in TMAs to encourage, and emphasise the need for, thinking about how you present your mathematics.

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Question 1 (Unit 8) – 16 marks

This question (and the next) start the assessment of the modelling strand of MST210. The aim of the modelling strand is to complete the modelling activity and write a modelling report, which is submitted as TMA 07. The questions here are designed to help you to write your modelling report by examining what makes a good report. To help you in this task of critically evaluating modelling reports, you are asked to assume the role of a tutor awarding marks.

Appendix 1 describes a modelling problem that is similar to the problem that you will consider for the modelling activity. Appendices 2 and 3 contain modelling reports that address this problem. Appendix 4 contains tutor notes for marking the modelling reports (this is an excerpt from the tutor notes for TMA 07).

(a) Award a mark (out of 5) for the **Specify the purpose** section of the report in Appendix 2. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*2*]

(b) Award a mark (out of 30) for the **Create the model** section of the report in Appendix 2. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*6*]

(c) Award a mark (out of 10) for the **Do the mathematics** section of the report in Appendix 2. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*4*]

(d) Award a mark (out of 10) for the **Interpret the results** section of the report in Appendix 2. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*4*]

Question 2 (Unit 8) – 16 marks

Your task in this question is the same as the previous question, but you are now asked to consider the second modelling report contained in Appendix 3.

(a) Award a mark (out of 5) for the **Specify the purpose** section of the report in Appendix 3. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*2*]

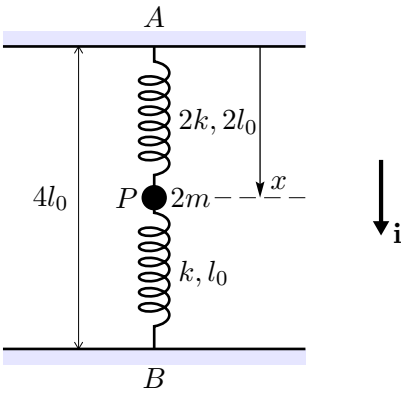
(b) Award a mark (out of 30) for the **Create the model** section of the report in Appendix 3. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*6*]

(c) Award a mark (out of 10) for the **Do the mathematics** section of the report in Appendix 3. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*4*]

(d) Award a mark (out of 10) for the **Interpret the results** section of the report in Appendix 3. Write a brief statement to justify the amount of marks you award in a style suitable for feedback to an MST210 student. [*4*]

Question 3 (Unit 9) – 15 marks

A particle P of mass $2m$ is attached to a model spring of natural length $2l_0$ and stiffness $2k$. The other end A of the spring is attached to a fixed point, and the spring hangs vertically downwards. A second spring of natural length l_0 and stiffness k is attached to the particle, with its other end being fixed at the point B at a vertical distance $4l_0$ below A, as shown in the following diagram.



The particle is displaced vertically and then released so that the system oscillates in one dimension. You may ignore air resistance in this question. Take the origin at A and the x-axis in the direction from A to B, as shown in the diagram.

(a) Draw a force diagram indicating all the forces acting on the particle, and briefly describe each force. [2]

(b) Copy and complete the following table to give the force due to each spring. The notation is the same as that used in the unit.

Spring	Spring length	Natural length	Extension	Stiffness	$\widehat{\mathbf{s}}$	**H**
AP		$2l_0$		$2k$		
BP		l_0		k		

Express all the other forces in component form. [5]

(c) Derive a differential equation of motion for the particle. [3]

(d) Find the equilibrium position of the particle. [2]

(e) Write down the general solution of the differential equation that you obtained in part (c). [2]

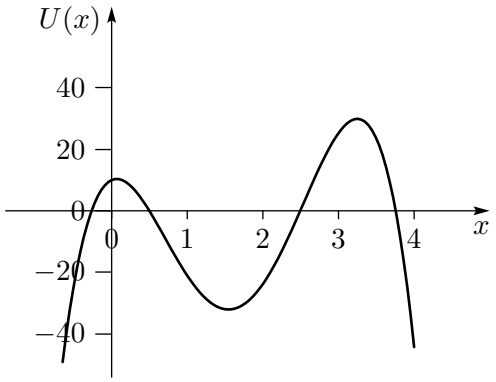
(f) Hence write down the period of the oscillations of the particle when it is in motion, in terms of the given parameters. [1]

Question 4 (Unit 9) – 5 marks

This question concerns a system with potential energy function given by

$$U(x) = -8x^4 + 52x^3 - 86x^2 + 11x + 10 \quad (-\infty < x < \infty),$$

whose graph is sketched below.



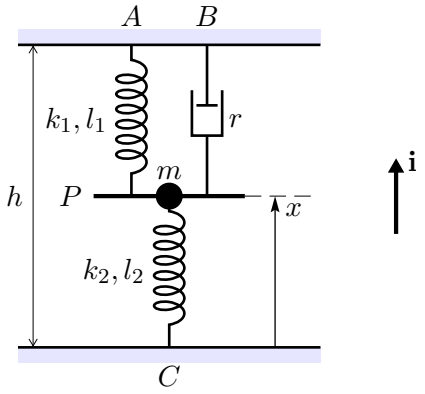
(a) Suppose that the total energy of the system is a constant, $E = -10$. Using Maxima, or otherwise, find the range or ranges of x-values (accurate to one decimal place) that could represent a motion of the system. Justify your answer by including an appropriate graph, or Maxima printout (such as your input to and the Maxima output from the command `find_root` or the command `allroots`), or details of your calculation. [*4*]

(b) Now suppose that the total energy is $E = 30$. Find the range or ranges of x-values (accurate to one decimal place) that could represent a motion of the system in this case. [*1*]

Question 5 (Unit 10) – 16 marks

The suspension of a modified baby bouncer is modelled by a model spring AP with stiffness k_1 and a model damper BP with damping coefficient r. The seat is tethered to the ground, and this tether is modelled by a second model spring PC with stiffness k_2. Model the combination of baby and seat as a particle of mass m at a point P that is a distance x above floor level.

The bouncer is suspended from a fixed support at a height h above the floor. The suspending spring has natural length l_1, while the tethering spring has natural length l_2. Take the origin at floor level, with the unit vector $\mathbf{i}$ pointing upwards, as shown in the following diagram.



(a) Draw a force diagram showing all forces acting on the particle P. [2]

(b) Copy and complete the following table, and hence write down the force on particle P due to each model spring. The notation is the same as that used in the unit.

Spring	Spring length	Natural length	Extension	Stiffness	$\widehat{\mathbf{s}}$	$\mathbf{H}$
AP		l_1		k_1		
CP		l_2		k_2		

[2]

(c) Copy and complete the following table, and hence write down the force on particle P due to the model damper. The notation is the same as that used in the unit.

Damper length	Rate of change of length	Damping constant	$\widehat{\mathbf{s}}$	$\mathbf{R}$
		r		

[2]

(d) Model the remaining forces acting on the particle P, and hence show that the equation of motion of the mass is

$$m\ddot{x} + r\dot{x} + (k_1 + k_2)x = k_1(h - l_1) + k_2 l_2 - mg.$$ [4]

(e) In SI units, suppose that $m = 2$, $k_1 = 90$, $k_2 = 38$, $r = 16$, $h = 2$, $l_1 = 0.75$ and $l_2 = 0.75$. Determine the natural angular frequency of the system to two decimal places.

Using the same data, calculate the damping ratio for this mechanical system to two decimal places, and hence state whether the system undergoes strong, weak or critical damping. Comment on whether this damping is appropriate for a baby bouncer. [6]

Question 6 (Unit 10) – 5 marks

Consider a particle that moves according to the equation of motion

$$5\ddot{x} + 20\dot{x} + 16x = 10\cos(\Omega t),$$

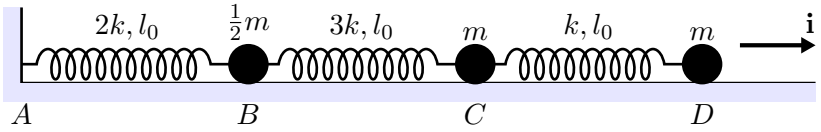
where Ω is a constant.

(a) Suppose that $\Omega = 4$. Calculate the amplitude of the resulting oscillations of the particle after a long time has elapsed. [2]

(b) Calculate the damping ratio for this mechanical system to two decimal places, and hence state whether the particle can undergo resonance. [3]

Question 7 (Unit 11) – 16 marks

Three particles, B, C and D, of masses $\frac{1}{2}m$, m and m, respectively, are connected to three springs as shown in the following diagram.



The left-hand spring is connected to a fixed point A, and the particles are constrained to move in a straight line on a smooth horizontal surface. Each spring has natural length l_0. Spring AB has stiffness $2k$, spring BC has stiffness $3k$, and spring CD has stiffness k. The particles are displaced from their equilibrium positions and released. The displacements of the particles B, C and D from their equilibrium positions are x_1, x_2 and x_3, respectively, and time t is measured from the instant when the particles are released.

(a) Draw a force diagram showing all the forces acting on each particle. [3]

(b) Copy and complete the following table to give the change in spring force from equilibrium for each spring acting on each particle. The notation is the same as that used in the unit.

Spring	Particle	Extension	Stiffness	$\widehat{\mathbf{s}}$	$\Delta\mathbf{H}$
AB	B				
BC	B				
BC	C				
CD	C				
CD	D				

[5]

(c) Hence derive the equation of motion for each particle, and show that in matrix form these equations may be expressed as

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = \frac{k}{m} \begin{pmatrix} -10 & 6 & 0 \\ 3 & -4 & 1 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

[8]

Question 8 (Unit 11) – 6 marks

The oscillations of three particles A, B and C are governed by the matrix equation

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} -1 & \frac{1}{2} & 0 \\ 1 & -3 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix},$$

where x, y and z are the particles' respective displacements from equilibrium.

(a) Show that the vector $(0.5 \quad -2.87 \quad 1)^T$ is an approximate eigenvector of the dynamic matrix (to two decimal place accuracy) by calculating the corresponding eigenvalue. [*2*]

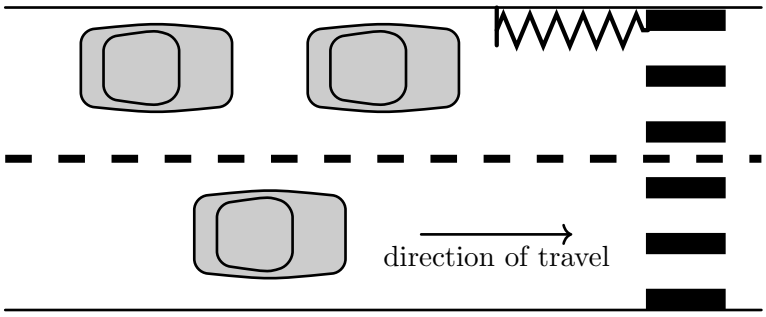
(b) Calculate the corresponding normal mode angular frequency for the normal mode with the given normal mode eigenvector. [*2*]

(c) Give an initial displacement vector that would lead to the normal mode motion corresponding to the given eigenvector. [*1*]

(d) For this normal mode motion, state (with a reason) which of the particles are moving in phase. [*1*]

Appendix 1: Modelling problem

The Open University main campus, in the UK, has a two-lane ring road. Several years ago, to create additional car parking space, most of the ring road was made one-way to allow parking in the left-hand lane, leaving just the right-hand lane for traffic. Usually this parking space is occupied, and as a result pedestrians crossing the road are at risk as they are obscured by parked cars. So pedestrian crossings are being considered for pedestrian safety. Parking will not be permitted near a crossing on the approach side, so that the driver of any approaching car will have a clear view of anyone waiting to cross; markings will be painted on the road, like the zigzag markings before pedestrian crossings that are found in many towns and cities, to indicate that no parking is allowed there under any circumstances, as shown in the diagram below.



The question to investigate is:

> **How far should the no-parking zone extend before a pedestrian crossing?**

There is still a shortage of parking space on the campus, so while the no-parking zones should be long enough to achieve their objective (of ensuring safety for pedestrians), they should not be any longer than is necessary.

(*Hint*: Exercise 8 of Unit 3 describes a stopping distance model that is used in the UK *Highway Code*.)

In an initial brainstorming session the following features were identified.

Length of no-parking zone	Stopping safely
Driver's view	Pedestrian safety
Speed limit	Size of parked car
Size of pedestrian	Weather conditions
Condition of road surface	Diameter of ring road

Appendix 2: Modelling report 1 – No-parking zone

Specify the purpose

The problem is to find how long a stretch of the left-hand lane on the approach to a pedestrian crossing, in which no parking is allowed, must be, in order that the distance of a car from the crossing when the whole of the crossing comes into the driver's view is sufficient for the car to stop safely before reaching the crossing, if it is necessary to do so.

Create the model

The plan is to find out the distance at which a car driver needs to see a pedestrian, and still be able to stop safely (using the hint). The distance will be affected by the field of view, which will be subject to any parked cars, and to the position of the driver relative to the kerb. A simple geometric argument establishes the point at which the driver can see the pedestrian, and stop safely before the zebra crossing.

Assumptions:

1. The road is of constant width, and the width of the left-hand lane is half the width of the road.
2. The parked vehicle nearest the crossing is a high-sided vehicle, and it is parked as close both to the crossing and to the centre of the road as it can be.
3. The approaching car is travelling as close to the centre of the road as it can.
4. The driver does not alter the position of his head from its normal position in order to try to see round the parked vehicle.
5. The minimum safe stopping distance is that determined by the Highway Code formula.
6. Drivers on the Open University campus ring road obey the speed limit, and this is the maximum speed at which they will travel.
7. The road is level.
8. The road can be assumed to be straight even though it is a ring road.
9. The pedestrians are at the nearest corner of the crossing, and have zero width.

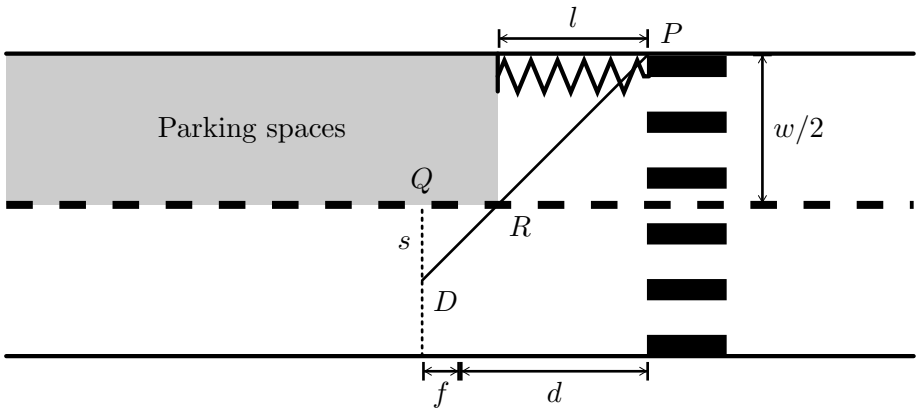
Variables:

d	the distance of the front of an approaching car from the crossing when the whole of the crossing comes into view	m
l	the length of the no-parking zone	m

Parameters:

w	the width of the road	m
s	the horizontal distance of the driver's head from the left-hand side of the car	m
f	the horizontal distance of the driver's head from the front of the car	m
$umax$	the maximum speed with which cars are driven on the ring road	$\mathrm{m\,s^{-1}}$

The sketch below shows the position of the approaching car at the instant that the driver can just see the left-hand end of the crossing, which is the instant when the whole of the crossing first comes into view. It incorporates Assumptions 1, 2, 3, 8 and 9. The driver is at the point marked D, and the pedestrian is at the point marked P. In order to calculate the distance of the front of the car from the crossing at this instant, we must find the distance between the corner of the no-parking zone, marked R (with R, P and D all on the one straight line), and the point on the left-hand side of the car level with the driver's head, marked Q (Assumption 4).



The width of the left-hand lane is $\frac{1}{2}w$ (Assumption 1). Thus, using similar triangles,

$$\frac{RQ}{s} = \frac{l}{w/2}. \tag{1}$$

The distance of the front of the car from the crossing is obtained by adding RQ to l and subtracting from the result the horizontal distance of the driver's head from the front of the car; so

$$d = l + RQ - f = l + \frac{2sl}{w} - f = l\left(1 + \frac{2s}{w}\right) - f, \tag{2}$$

having substituted for RQ using Equation 1.

Do the mathematics

The distance of the car from the crossing at the instant when the whole of the crossing comes into view (Assumption 9) must be sufficient to allow the car to stop safely before the crossing if necessary. Because of Assumptions 7 and 5, the distance is determined by the Highway Code formula, and assuming that the speed of the approaching car is the maximum attained on

the ring road (Assumption 6), then

$$d = 0.67umax + \frac{umax^2}{13.1}, \tag{3}$$

as given in the solution to Exercise 8 of Unit 3 (using the hint). Eliminating d from Equations 2 and 3 gives

$$l\left(1 + \frac{2s}{w}\right) - f = 0.67umax + \frac{umax^2}{13.1}, \tag{4}$$

and so, rearranging Equation 4 to make l the subject gives

$$l = \frac{f + 0.67umax + umax^2/13.1}{1 + 2s/w}. \tag{5}$$

This formula gives the length of the no-parking zone as a function of the speed limit.

Interpret the results

To find a numerical value for l, numerical values of f and s, which can be found by measurement (of any car), are required. However, where there is a choice, considerations of safety require the choice of values that will make l largest; that is to say, the values of f and s for a long thin car will be more suitable than those for a short fat one.

Take $f = 1.5$ and $s = 1$.

The average width is 5.5 m, so $w = 5.5$. With $umax = 6.7$ (Assumption 6), $l = 6.96$.

Thus, the model suggests that the no-parking zone should be 7 m long (rounding up to the nearest metre).

The model can also predict the length of the no-parking zone for other speed limits, as given in the following table (rounding to three decimal places).

Speed limit	$umax$	l
10	4.469	4.452
20	8.939	10.038
30	13.408	17.858

These predictions of the model can be compared with measured lengths of no-parking zones for roads with different speed limits.

Appendix 3: Modelling report 2 – No-parking zone

Specify the purpose

The aim of the model is to derive a minimum safe distance for the length of the no-parking zone that precedes a pedestrian crossing on a road with a specified speed limit.

Create the model

The model will consider a worst-case scenario where a pedestrian is about to cross the crossing and is hidden from the approaching driver as far as possible. At the point that the pedestrian becomes visible to the driver, they will brake and the stopping distance should be less than the distance to the crossing.

Assumptions

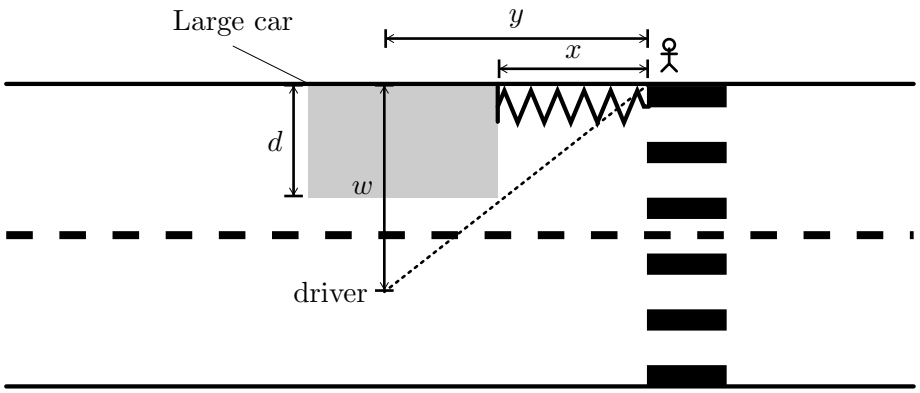
1) The road is straight with the kerbs parallel.
2) The road has dry conditions with good visibility.
3) There is a large car blocking visibility parked adjacent to the no-park zone.
4) A driver will brake as soon as possible after a pedestrian is seen.
5) A driver will drive at the speed limit of the road.

Variables and parameters

Quantity	Symbol	Units
Length of no-park zone (see diagram below)	x	m
Perpendicular distance of driver from kerb (see diagram below)	w	m
Speed limit for road	v	m/s
Width of car obstructing the view (see diagram below)	d	m
Distance car needs to stop when travelling at the speed limit	y	m
Reaction time for driver	t_R	s
Magnitude of acceleration of car when driver applies brakes	a	m/s^2

By assumption 1 the road is straight, as in the diagram below.

By assumption 2 a large car is parked adjacent to the no-park zone, as in the diagram below.



From the diagram we have two similar right-angled triangles. One of the triangles is formed between the driver, the closest point on the kerb and the pedestrian. The other triangle is formed between the corner of the parked car, the closest point on the kerb and the pedestrian. So corresponding sides are in proportion

$$\frac{x}{d} = \frac{y}{w}.$$

The stopping distance model used in the Highway Code is described in Exercise 8 on page 190 of Unit 3. This model can be expressed as

$$y = t_R v + \frac{v^2}{2a}.$$

This formula is made up of two terms. The first term is the distance travelled while the driver is reacting, and the second term is the distance travelled while braking.

This model applies here by assumptions 1 and 2.

The reaction time starts immediately the driver sees the pedestrian by assumption 4.

The speed v used in the stopping distance model is the speed limit of the road by assumption 5.

Do the mathematics

The relationship between x and y is rearranged to make x the subject to give

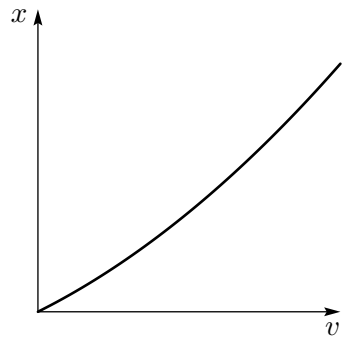
$$x = \frac{d}{w}\, y.$$

Now substitute for the distance y using the stopping distance model to obtain

$$x = \frac{d}{w}\left(t_R v + \frac{v^2}{2a}\right).$$

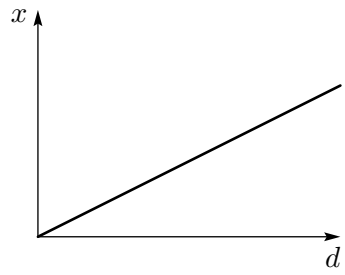
This is the desired model for the length of the no-park zone x in terms of known variables and parameters.

The distance x is a quadratic function of the speed limit v and a typical graph is given by



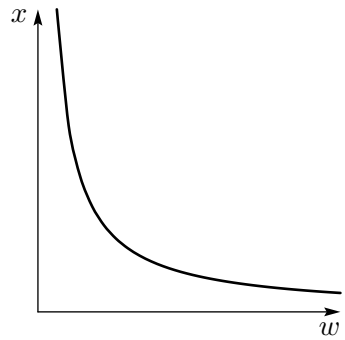
This graph looks reasonable as it goes through the origin (as if speed limit is very small then the no-park length can also be small) and x increases with increasing v.

The distance x is a linear function of the width of the parked vehicle d, as shown below.



Again this is reasonable since if $d = 0$ then the parked car does not obstruct the driver's view and there is no need for a no-park zone (so $x = 0$ is the optimal distance). The wider the vehicle, the longer the no-park length should be.

The distance x is inversely proportional to the distance of the driver from the kerb w, as in the graph shown below.



This again fits in with intuition as if w is very large then the no-park length can be very short and the driver would still be able to see the pedestrian in time.

Applying the method of dimensional analysis to the model gives

$$[x] = \frac{[d]}{[w]}\left([t_R][v] + \frac{[v^2]}{[a]}\right).$$

$$L = \frac{L}{L}\left(T\,LT^{-1} + \frac{L^2T^2}{LT^{-2}}\right).$$

$$L = L + L.$$

As all terms have the dimensions of length, the model is dimensionally consistent.

Interpret the results

It is better to choose a large vehicle for the parked car as this will give the worst-case scenario. Looking up the width of a Ford Transit van on the internet gives a width of 2.1 m.

Measuring a typical distance of a driver from the far kerb while driving gives the distance 3 m.

For the highway code model of stopping distances, the parameters are given in Exercise 8 of Unit 3. The thinking time is 0.67 s and the magnitude of the acceleration during braking is approximately $\frac{2}{3}g \simeq 6.5\,\mathrm{m\,s^{-2}}$.

The Open University ring road has a speed limit of 20 mph, which gives a stopping distance of $y = 12$ m (using Table 2 on page 191 of Unit 3).

This gives a no-park length

$$x = \frac{2.1}{3} \times 12 \simeq 8.4.$$

This is the prediction of the model. The suggested length of the no-park zone is 8 metres (to the nearest metre).

The model predicts the following lengths for no-park zones for roads with different speed limits.

Speed limit/mph	No-park zone length/m
20	8
30	15
40	24
50	35

Appendix 4: Tutor notes for modelling report

These notes are an excerpt from the tutor notes for TMA 07 that your tutor will use to mark your modelling report. These tutor notes relate to the first four sections of the modelling report, which leads to the creation of a first model for the problem (as described in Unit 8).

Specify the purpose **5 marks**

Define the specific problem to be solved. Write a clear, succinct statement of the specific problem addressed in your report, in your own words (do not just repeat the given problem statement).

Comments on the allocation of the marks etc.

- *Is there repetition? Deduct up to 1 mark for significant repetition.*
- *Is there vagueness? Deduct up to 1 mark for significant vagueness.*
- *Are the main points covered in some way? Deduct 1 mark for each omission (but do not penalise if it is badly written as this may be covered by the marks for vagueness and repetition).*
- *Are there any points in the specification that are not investigated in the report? Deduct 1 mark for each feature mentioned that is not investigated further.*
- *Does the specification inform you of what is the problem to be solved? Deduct up to 1 mark if it does not.*
- *This should not be a copy of the wording in the TMA; deduct 2 marks if it is a direct copy.*
- *Shorter is usually better. Deduct 1 mark if significantly over length (say more than 100 words).*

Create the model

Describe features investigated and outline mathematics used **5 marks**

Mention the features from your initial feature list that you are investigating. Give a qualitative description of the approach to be used in the first model, to explain why and how the first model will be formulated.

Comments on the allocation of the marks etc.

- *Are all features listed? Deduct 1 mark for each feature of the problem that is used but not listed.*
- *Does the outline inform you of the approach that is to be taken in the formulation? Does the outline explain adequately what will come next? (You may have to read the actual formulation before deciding whether this outline does explain the approach taken.)*
- *The outline should not contain any mathematical notation, should be about 100 words long, and not longer than 150 words.*
- *It is not necessary to state what is not being considered, but it is sometimes helpful.*

State assumptions **7 marks**

Create a numbered list of clearly stated assumptions used in the model (take care not to miss assumptions or include assumptions that are never used). Do not attempt to justify assumptions here.

Data values should not appear in assumptions, so, for example, 'the width of the road is 10 m' should be replaced by 'the width of the road is constant'.

Comments on the allocation of the marks etc.

- *This is perhaps the part of the report that may show the greatest variation, and therefore the most difficult to assess.*
- *As this problem concerns safety, the assumptions should be about the worst case that will be met and not about what happens in general. It is sensible when creating a model to try to imagine the worst situation that can arise (within reason), because then normal situations will be automatically covered. A model that considers a typical situation is not necessarily a bad model, so this may not merit deducting marks but should be commented upon.*
- *Are the assumptions realistic? For example, it is not realistic to assume that the driver applies the brakes the instant the pedestrian is seen.*
- *Even if the assumptions are not wholly realistic, is it possible that they do approximate to reality?*
- *For this problem, there should be assumptions about:*
  - *the position of the moving car or its driver in relation to the kerb;*
  - *the parked car and its position;*
  - *the road being straight;*
  - *being able to use the Highway Code formula;*
  - *the position of the pedestrian at the kerb.*

  Deduct 1 mark for each omission.

Choose variables and parameters **5 marks**

Create a table of all symbols used in the model. For each symbol, state a clear definition and its associated units. It is not necessary to distinguish between variables and parameters.

Comments on the allocation of the marks etc.

- *Deduct 1 mark, with an upper limit of 2 marks, for each definition in which the units are not specified.*
- *Are the definitions vague or specific? Deduct up to 2 marks if the definitions are of the type 'length', 'time' without further elaboration; the definition of variables such as these needs to be better specified, e.g. 'length of no-parking zone', 'time since the pedestrian appeared at the kerb'.*
- *Are all the variables and parameters used in the report defined? Deduct 1 mark if there is more than one variable or parameter that is not used in the report, or there is a parameter that is used but not defined.*
- *Although the distinction between variables and parameters is useful, there is no necessity to distinguish them.*
- *Are the symbols used for the parameters and variables helpful? Do they have suffixes that are confusing? Deduct 1 mark if the variable names are composed of two or more characters following each other (e.g. 'um' for maximum speed, instead of 'u_m').*
- *It is helpful to have the same type of variable (e.g. lengths)*

grouped together in the definitions, perhaps with the same letter, but with different subscripts to distinguish between them.

- *It is recommended that the usual mathematical symbols (e.g. 'g', 'e', etc.) are not used as variable or parameter names. Deduct half a mark if such a symbol is used.*
- *Using SI units is preferred. If SI units are not used then more attention should be paid to the conversion factors used in the formulation, solution and interpretation of the model. It is better to include conversion factors as parameters, rather than as values. Comment on this, but do not deduct marks (it may lead to errors elsewhere, however, that will lose marks).*

Formulate mathematical relationships **13 marks**

Derive relationships between your variables and parameters. You should explain how the equations follow from your assumptions, which should be referenced.

Comments on the allocation of the marks etc.

- *Note how the steps of the derivation are linked to the assumptions. Each assumption should be used somewhere in the report, otherwise it is superfluous. Deduct 1 mark in this section for each assumption to which there is no reference, up to a maximum of 4 marks (however, assumptions may be used for the first time in the next section, so you should consider the assumptions over the whole report).*
- *It is not always clear whether a part of the modelling process should be included here or in the next section; there is some overlap between the two sections. The use of the section headings is to help structure the report, and there should be no penalty for having some of the derivation in the next section.*
- *Note how useful a diagram is to explain distances and positions; if no diagram is supplied, then please emphasise the importance of including one. Do not deduct marks if the distances are sufficiently well-explained without the use of a diagram.*
- *Does the formulation follow from the assumptions, and is it as described in the outline?*
- *Using numerical values here is bad modelling practice. Deduct up to 5 marks for this. Using variables and parameters generalises the model, and then the model can be applied to many other situations.*
- *Note that it is the modelling process, and not the model, that is being assessed here. The model can be as simple as that given here, or simpler, with adequate simplifying assumptions.*
- *An explanation of what is occurring at each stage should be included; up to 4 marks can be deducted if there is not enough adequate explanation.*

Do the mathematics **10 marks**

Derive a first model. Solve your first model to find the variable of interest (as specified in the purpose of the model) in terms of other variables and parameters. Clearly state the mathematical model

derived. It is not necessary to have one overall explicit equation; it is possible to have a series of equations, which may aid clarity, or an implicit equation (that will be solved numerically). Your solution at this stage should not include particular data values.

Draw graphs showing typical relationships. Sketch graphs to show the expected variation of variables predicted by your model. Use typical values for any parameters.

Check your model using dimensional analysis.

Comments on the allocation of the marks etc.

- *The formulas, and their derivation, should be fairly simple, depending on the modelling assumptions.*
- *Here again the introduction of numerical values should be discouraged. The question asks for a formula involving the length of the no-parking zone as a function of the maximum car speed, and other parameters; deduct up to 3 marks if numerical values have been used instead at this stage.*
- *The derivation should include references to the assumptions or equations on which it is based. Up to 2 marks can be deducted for absence of references to the assumptions or equations.*

Interpret the results **10 marks**

Collect relevant data for parameter values. Usually relevant data is available on the internet (or in the library), in which case the source should be referenced. For some models it is easier to perform a simple experiment, in which case the deduced parameter values should be stated here and the experiment should be described in an appendix.

Describe the mathematical solution. Substitute data into your model to find a solution. Clearly state in words this solution and how it relates to the purpose of the model. This should be written in a form that could be understood by a lay-person, by presenting it, for example, as a set of instructions, a graph or a table.

Find predictions to compare with reality. Look for any predictions of your model, or part of your model, or a corollary of your model that may be tested.

Comments on the allocation of the marks etc.

- *It is not necessary to use the worst-case values as in the model solution, although it is preferable; for reasons of safety, it is best to use more extreme values rather than average values. Do not deduct any marks for this.*
- *The average of the widths has been used here; it may be prudent to use the value of the narrowest width of the road, being the worst case.*
- *It is useful to structure the presentation of the values, but do not deduct any marks if the data are presented randomly, just recommend a structure to make it easier to follow. Remember that different assumptions may lead to slightly different results.*
- *The values given in the final equations should not be quoted to more than three significant figures. Deduct 1 mark for quoting an excessive number of significant figures, not warranted by the model.*